Rereading Jean Lave 30 years on
Analogy and transfer-in-pieces

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ABSTRACT

This paper explores and evaluates some of the criticisms of a cognitive approach to learning leveled by Lave in *Cognition in Practice* (1988). The paper progresses by initially identifying learning transfer as the focal topic of Lave's work. Two lines of criticism are identified, one called practice-based and one called concept-based. Through a renewed analysis of one of the empirical cases reported by Lave, I set out to show how this empirical material did not lend itself to Lave's conclusions, in so far as these conclusions rejected central tenets of cognitive theories of learning. Concerning the concept-based line of criticism, I show how the phenomenon of analogically related uses of words plays a role in defusing one version of the concept-based line of criticism. Concerning the practice-based criticism, I show how an oft-overlooked form of analogical inference, coupled with elements of a cognitive, transfer-in-pieces approach, offers a better account of Lave's case than the one she offered. The paper concludes with reflections on the lasting significance of Lave's work.

Keywords: *transfer, situated learning, structure mapping, analogy, logic*

1. Introduction: Lave's criticism of transfer studies

The year 2018 marked the 30th anniversary of the publication of Jean Lave's *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life.* The book has been highly influential in shaping the study of learning, and Lave's work was central to the development of situated learning theory. Large parts of the learning sciences are still responding to, and exploring some of Lave's criticisms of learning transfer. One way of paying tribute to great works is by disagreeing with them. This paper identifies and critically discusses two lines of argument found in Lave's book. They both challenge

the idea that learning transfer is a key phenomenon when seeking an understanding of learning. The first line of argument, which I call "concept-based", is inspired by wider themes in the philosophy of language. Influential commentators have seen these themes elaborated in Lave's work and appeal to them in expositions of it. The other line of argument, which I call "practice-based", derives from the data that Lave brought to bear on the question of learning transfer. This paper seeks to counter the concept-based argument against transfer, and instead offers an alternative interpretation of one of Lave's cases. This alternative is argued to have greater explanatory power, and relies on elements of a cognitive approach to learning: analogy and transfer-in-pieces.

In her book, Lave defends several, closely related claims: firstly, that there is a significant gap between everyday, "street" math, conducted by what she calls "just plain folk", and school math; secondly, that scientifically informed school math is widely and wrongly assumed to carry greater value than everyday, practical math; and finally, that the ethnographic studies she relied on demonstrate that when one moves beyond carefully constructed "toy tasks" in transfer research, little transfer from classroom instruction occurs. Lave's criticism gained strength in the wake of ethnographic studies of mathematical skills in various, non-school settings, conducted by Lave and others during a stretch of approximately fifteen years prior to the publication of *Cognition in Practice.* A great deal of discussion on learning transfer was prompted by its publication, sparking a lively debate about the relative merits of a cognitive approach to learning (e.g. Anderson et al., 1996, 1997) and alternative perspectives (e.g. Greeno, 1997).

Of the three related issues, transfer of learning appears to be the most widely discussed topic. In some ways of thinking about learning, the concepts of learning and transfer are hard to separate. The word "transfer" suggests a carrying of something from one place to another. Since no two places – or more generally, contexts or situations – are entirely the same, one could say that "all learning is transfer of learning" (Haskell, 2000, p. 24). Perkins (2009) recounts students having been instructed to calculate the time it takes a ball to reach the ground from a 100-meter tall tower. When asked to calculate the time it takes for a ball to reach the bottom of 50-meter well, some students complained that they had not been taught "well problems". Their instruction had failed to result in both transfer and learning. The notion of transfer has long been the subject of intense scrutiny in the learning sciences, with several special issues of, and strands in, journals dedicated to the topic (Engle, 2012; Goldstone & Day, 2012; Lobato, 2006; Segers & Gegenfurtner, 2013) as well as cases of possible transfer regularly being the subject of investigation (e.g. Sala & Gobet, 2016).

In spite of the influential criticism leveled by Lave, studies of transfer based on structure mapping persist (Gentner, 1983, 1989; Reed, 2012; Wagner, 2010), as do studies that insist on a close connection between analogical reasoning and transfer, and which have sought to refine our understanding of the underlying mechanisms of effective transfer (Alexander & Murphy, 1999; Holyoak & Thagard, 1989; Kapon & diSessa, 2012). Cognitive approaches to instruction, learning and transfer exhibit

great variety. For the purposes of the present paper, I group them together based on their commitment to the following view: transfer is possible through a process that involves abstracting a formal, schematic form of knowledge from a (range of) particular example(s) and applying this schematised knowledge in a context different from any of the original ones where instruction took place. An example would be to learn about a logical fallacy such as *affirming the consequent* (From "if P then Q", and "Q", one cannot validly infer "P"), and then recognising the fallacy in a range of everyday and scientific reasoning contexts. Another example, explored by Gentner, Loewenstein & Thompson (2003), is negotiation strategies, such as "trade-off" and "contingent contract", being applied in an analysis of a range of negotiation scenarios. Abstracting and generalising are offered as key elements in the account of transfer advanced by Lobato (2008), and it is readily targeted by those who criticise the notion of transfer and studies of it. Under the banners of a variety of theories, most notably situated learning theory, there has been a widespread turn against utilising notions of generality and abstraction in connection with discussions of learning and transfer (Sawyer & Greeno, 2008).

Lave believed that her data demonstrated that hardly any remnants of instruction were discernable in everyday use of mathematics. This approach has informed the argumentative strategy of others who are critical of transfer. An example is Carraher & Schliemann: "We will attempt to point out instances where [middle school students'] learning is influenced by what *they already know*. We hope to show why a theory of transfer cannot provide a solid foundation for explaining such examples of learning" (2002, p. 2 my emphasis). According to this line of reasoning, if a given theory of transfer cannot account for purported, clear cases of influence by what learners "already know", so much the worse for the theory. A key element in this strategy becomes the notion of what the learner "already knows", as our conception of this deeply influences what we can look for as having been transferred. The knowledge-in-pieces approach (diSessa, 1988, 1993) and subsequently, the transfer-in-pieces approach (diSessa & Wagner, 2005; Wagner, 2006, 2010) meet the learning transfer challenge, by minutely identifying the existing knowledge elements and structures that serve students in gradually interpreting and learning about new phenomena by relying, among other things, on structures apprehended during instruction.

Below, I discuss one of Lave's cases of a practice-based argument against learning transfer. After a brief exposition of the concept of analogy, I show how employing a cognitive approach to learning transfer, inspired by Wagner (2010), can offer a more adequate analysis of Lave's own data. It is also through an appeal to analogy that a concept-based line of criticism is discussed. Again, an understanding of analogy features centrally, but in this case, analogical use of words rather than analogical argument is pivotal to the analysis.

In addition to applying the transfer-in-pieces approach to a case of "everyday practice mathematics", the analysis offered below points to an oft-overlooked aspect of an element of structure mapping called "inference". Transfer and learning, like

most complex cognitive processes, is likely to be accounted for by an appeal to more basic processes. Gentner et al. (1993) registered a consensus on there being six sub-processes in similarity based transfer theories. Accessing an analogy is the first, and making an inference is the third process. This paper highlights one way the third process can be actualised: through deductive, rather than inductive, inference.

2. Practice-based and concept-based criticisms of transfer

Lave posed the question of transfer in an admirably clear fashion. This section explores the way she, along with sympathetic interpreters of her work, have responded to it. Here is an example of commentary on mathematics in practice, with mention of the mathematical instructional practices that Lave could not identify in the data:

> … essentially no problem in store or kitchen was solved in school algorithmic form. Transformational rules (which eliminate algorithmic approaches to fractions and decimals) do not travel, nor does place holding notation, since paper and pencil are not used, calculus, trigonometry, analytic geometry, algebra etc etc. The question really should be "is there anything that *does* transfer?"
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> *(Lave, 1988, p. 199)*

Lave's negative response was based on interesting and then novel empirical material gathered from a range of ethnographic studies of mathematical practice conducted in various cultures (e.g. Herndon, 1971; Lave, 1977a, 1977b, 1982; Scribner & Fahrmeier, 1982). From a North American context, the Adult Math Project brought studies of the mathematical practices of e.g. supermarket shoppers and dieters to the discussion of learning. Let us recount one of the cases of what Lave called just plain folk mathematical reasoning, which was based on the unpublished doctoral work of de la Rocha (1986). A dieter has been challenged to calculate a specified amount of cottage cheese for a recipe:

> In this case they were to fix a serving of cottage cheese, supposing the amount laid out for the meal was three-quarters of the two-thirds cup the program allowed. The problem solver in this example began the task muttering that he had taken a calculus course in college … Then after a pause he suddenly announced that he had "got it!" From then on he appeared certain he was correct, even before carrying out the procedure. He filled a measuring-cup two-thirds full of cottage cheese, dumped it out on the cutting board, patted it into a circle, marked a cross on it, scooped away one quadrant, and served the rest. Thus, "take three-quarters of two-thirds of a cup of cottage cheese" was not just the problem statement but also the solution to the problem and the procedure for solving it. The setting was part of the calculating process and the solution was simply the problem statement, enacted with the setting.
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> *(Lave, 1988, p. 165)*

The dieter wished to use three quarters of the two thirds of a cup prescribed by the dieting program. Not being equipped with material tools (calculator or pen and pencil) or some other canonical method for solving a piece of pure arithmetic ($2/3 \times 3/4$), the dieter emptied a two-thirds full cup, shaped the contents into a circle, made a cross, and scooped a quadrant away, in this way arriving at a result that the dieter felt confident about once the procedure was conceived, but before it was carried out.

Lave used such examples of mathematics-in-practice to support a view of learning that emphasises situativity, thereby challenging a view of learning that underscores instruction and transfer of "decontextualised", abstract knowledge. Such cases were meant to convince us that in general, learning activity can only be explained in relation to its social and material context. Others have suggested that, concerning the dieter, the solution "... reflected the nature of the activity, the resources available, and the sort of resolution required in a way that problem solving that relies on abstracted knowledge cannot." (Brown, Collins, & Duguid, 1989, p. 35). Lave herself stressed how "'knowing what one is doing' is possible within a field for action, in activity in context" (1988, p. 165) and she chose to explore alternative characterisations of prerequisites for action − characterisations in stark contrast to abstract objects and processes such as transferable scripts and inference rules.

Lave took aim at the studies of analogy and transfer initiated by Reed, Ernst and Banerji (1974) and continued by Gick & Holyoak (1980) and Gentner et al (1983), finding what she took to be canonical, prescriptive forms of understanding, studied in lab-like contexts, to be incompatible with her studies of the practice of mathematics. Many seem to have accepted her analysis of the data and the conclusions she drew from it, in so far as they questioned widespread ideas about transfer. For example, in his reconceptualisation of transfer, Billett suggests that Lave's studies demonstrate how students fail "to make associations with what they know" (2013, p. 6).

Alongside this practice-based line of skepticism, a concept-based line of criticism has arisen, and frequently starts with interpretations of Lave's work. Several theorists within the tradition of situated learning draw on studies of language in their concept-based criticism. For example, the widely cited paper by Brown, Collins & Duguid mentioned above, which was the first to use the term "situated cognition" in discussions of research on learning, claims to be "deeply indebted to her groundbreaking work" (1989, p. 41). They rely on theories from the philosophy of language and empirical studies of children learning words, in order to formulate their version of a Lave-inspired criticism of transfer. Specifically, they draw on reflections on language offered by Miller and Gildea (1987), who confront the supposedly misguided views that children learn words and sentences as self-contained pieces of knowledge, contrasting such views with indexicals (such as "I" and "now"), whose reference is settled by the context of the utterance. Brown, Collins & Duguid (1989) concur with Miller & Gildea's viewpoint, adding: "Indexicals are not merely context-sensitive; they are completely context-dependent ... surprisingly, all words can be seen as at least partially indexical. All knowledge is, we believe, like language. Its constituent parts index

the world and so are inextricably a product of the activity and situations in which they are produced" (Brown et al 1989, p. 33). Later, Packer (2001), in an influential discussion of transfer in which Lave's book plays a key role, also relies on theories from the philosophy of language and science in order to represent the crux of Lave's criticism. He suggests that "Lave's (1988) view is not so much that performance is different in different settings, for that would imply an enduring underlying competence, but rather that qualitatively different arithmetical activities are at work here; different and distinct «language games,» to use Wittgenstein's (1953) term. And if these different activities are indeed incommensurate, how could there be transfer among them?" (Packer, 2001, p. 499).

Lave herself relied more on theories from philosophy of mind and cognitive science than studies of language, with her main target being what she called functionalism in cognitive science. Yet, she appears at ease with philosophy of language in her discussion on the importance and conception of context: "Rommetveit (1988) reminds us of «the intuitive appeal of pervading pretheoretical notions such as, for instance (Goffman, 1976, p. 303), 'the common sense notion … that the word *in isolation* will have a general basic, or most down-to-earth meaning.' Such presuppositions seem to form part of the myth of literal meaning in our highly literate societies.»" (Lave, 1993, p. 23).

Exactly what role these reflections on theories of language – and *inter alia*, the concept-based criticism – have played in the reception and interpretation of Lave's book and its interplay with situated learning is difficult to establish. At least two positions are possible: Drawing on philosophy of language has purely served as an expository aid. In cases where the positions presented (for example, when Brown et al rely on a particular understanding of the semantics of indexicals) fail, they only fail as ways of throwing light on key tenets of Lave's work, while leaving those tenets unscathed. The other position is that, for example, Packer's appeals to language games indeed capture central ideas in Lave's book, and *inter alia*, that any discussion of the soundness of these standpoints in philosophy of language is also a discussion of central strands in Lave's book. Determining which position concerning the effects of her work is nearer to the truth would mean engaging thoroughly with longstanding issues in philosophy of language: the semantics of indexicals, the success of Wittgenstein's later appeal to language games as a way of confronting his earlier work on logic, and the distinction between the literal and the metaphorical. No such attempt shall be made here, except for the discussion of incommensurability below.[1] Meanwhile, this concept-based criticism of transfer suggests that it is not only philosophy of mind discussions that are analogous to discussions in situated learning, as Sawyer & Greeno urge (2008). Philos-

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1   I believe the appeals to issues in philosophy of language enumerated in the previous paragraphs are at best radically incomplete attempts at establishing what they seek to establish, but I cannot here establish this claim. In Hansen (2010), I discuss the widespread acceptance of Wittgenstein's appeal to language games as a successful way of arguing against the existence of universal, logical forms, which in turn can be seen as central to a broadly cognitivist approach to learning.

ophy of language is frequently drawn on when offering an exposition and discussion of Lave's book and the tradition it helped form.

3. Responding to the transfer challenges through an understanding of analogy

Ultimately, Lave presents a thorough case for skepticism concerning learning transfer. To set up a response to Lave's arguments, this section offers a brief account of analogy. Responses to skepticism can be categorised as straight or skeptical solutions (Kripke, 1982). Skeptical solutions accept the conclusions of the skeptic – in the manner of Billett, cited above – and then proceed to attempt to accommodate the conclusions made, with a view to offering some altered account of the subject at hand. Straight solutions confront the sceptic directly, as it were, by seeking to point out why we should not accept the skeptic's conclusions in the first place. Responses to Lave's criticism of transfer tend to be skeptical solutions, where the concept of transfer is relinquished in favor of concepts like *transformation* (Lave, 1988), *consequential transition* (Beach, 1999), and *a set of dispositions with the learner* (Bereiter, 1995). While not offering a full account of learning, Lave and Wenger's notion of *participation in communities of practice* (Lave & Wenger, 1991) also focuses on social context in theories of learning, at the expense of a cognitive approach.

While not responding directly to the Lave's challenge, straight responses to the challenge of accounting for transfer can be seen in an insistence on providing more detailed data in connection with studies of learning, in order to obtain the learner's perspective, often from in-depth learning interviews (Kapon & diSessa, 2012; Lobato, 2008; Wagner, 2006). Another avenue taken in straight responses to the skepticism is a call to refine our understanding of the process of relying on analogy in learning (Clement, 1993; Holyoak et al., 2010; Holyoak & Thagard, 1989), in so far as analogy is assumed to be part of the learning transfer process. Finally, as Greiffenhagen & Sharrock (2008) effectively argue and Reed (2012) also suggests, a more detailed and alternate analysis of Lave's data may be called for as a way of responding to the skepticism. Not having had access to detailed data from Lave's cases, the straight solution offered in the following section relies on the two latter strategies: adding to our understanding of analogy and in light of that, offering a more detailed analysis. In what follows, I shall emphasise how we clearly *do* manage to identify identical conceptual material and *inter alia*, generality, across widely different contexts – a hallmark of logic as well as analogical language and reasoning.

In order to offer an alternative analysis of Lave's dieter, and at the same time confront Packer's version of concept-based transfer criticism, I now offer a brief overview of analogy. Analogy is widely appealed to in discussions on learning, and I offer this outline solely to highlight one kind of analogical inference, and one kind of word use that relies on analogy.[2] There are indeed a number of distinct uses of analogy and

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2   My account of analogy relies primarily on the published and unpublished writings of White (n.d., 2010), as well as van der Waerden (1961) and Bartha (2010).

forms of analogous arguments in existence. The background for using analogy is found in arithmetical operations that also concerned Lave. The concept of analogy was originally developed to overcome the challenge of incommensurability that Packer utilises in his description of Lave's position concerning transfer.

To give the relative magnitude of any two lengths, A and B, one offers two numbers, m and n, so that mA = nB. Rather than giving a rational number (m/n), the formula is interpreted geometrically: extend A m times and the length will be equal to *some* extension, n, of B in whole numbers. This interpretation integrates natural number theory with a theory of magnitude, putting mathematics at the core of a description of the universe. However, the existence of incommensurable lengths meant a crisis to the theory of relative magnitude: there are lengths, described and calculated in Pythagoras' theorem, so that for a given mA, no nB exists, and so, it appears possible to have a line with no determinable magnitude.[3] An account of relative magnitude that could account for both commensurable and incommensurable lengths was badly needed to avert this scientific crisis.

The answer to this challenge was the extremely fertile formula *A is to B as C is to D* (A : B :: C : D), developed by Theaetetus and Eudoxus.[4] When A and B are not directly comparable, their relative magnitude can be compared to that of another pair of lengths (equal, greater or smaller), and according to both Euclid and subsequently Aristotle, analogy is the equality of ratios. While originally at home in mathematics, Plato and in particular Aristotle would apply this formula beyond its mathematical confines to offer an account where, more generally, one can use the formula to compare things that cannot otherwise be directly compared – that are "remote in kind". In this way, "is to" is no longer purely a matter of a relationship between lengths, but a wide array of relationships, such as "gloves are to hands as socks are to feet". Aristotle used analogy most fruitfully in biology. Wings on a butterfly and a bird are morphologically quite different, but as Aristotle suggests, related by analogy. Analogy served as the key conceptual tool in discovering why animals have the parts they do. In his comparative biology, parts of animals differ by more or less, or by analogy. The lungs of a cat are larger than those of a a dog, but little else than size is needed to account for this difference. Studying fish, there are analogous relations with other animals, so that for example "skeleton : dog :: ? : fish", leaves "bones" identical in different organisms by analogy. Aristotle also suggested that answers to political questions like "What is *fair* payment to someone who teaches compared to someone who produces corn?" draw on an analogical relationship between otherwise incomparable amounts

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3   For a proof of the existence of incommensurable lengths, textbook material is available here: http://www.learner.org/courses/mathilluminated/units/3/textbook/03.php

4   I follow Roger White in concentrating on the solution Eudoxus is likely to have put forward. It is found in Euclid Books V and VI and Aristotle is likely to have had it in mind when speaking of analogy in mathematics.

of wisdom and corn. Finally, he pointed to analogy as underlying effective metaphors.[5] Like the mathematical notions of incommensurability widely used by Aristotle, he also derived mathematically inspired solutions.

Leaving Greek mathematics aside, and returning to Packer's framing of Lave's thought, we note that incommensurability is the order of the day in basic language use. While the question "*Which is longer, a marathon or Les Misérables?*" might be singled out for its lack of clear meaning, it also highlights the fact that we readily use the same word, "long", about very different things (runs and novels) and frequently transcend all sorts of boundaries between seemingly unrelated domains with our concepts. There is little in common between a running event and a novel, but we confidently use the word "long" for both, based on the analogous relation "*Les Misérables*: average novel :: marathon : average run". "Long" here is used in different contexts, with *apparently* nothing in common, but few would make the suggestion that "long" has particular indexical features or should be distinguished according to the context in which it is used.[6] "Calm" is another example discussed by Aristotle. It describes meteorological phenomena, people and music. Words such as these appear to traverse seemingly unrelated domains that seem difficult to measure against one another. There are more ways of construing Packer's appeal to activities, language games and incommensurability, but in so far as his construal of the concept-based case against transfer relies on themes in philosophy of language, the case appears weakened. Put differently, even if there were elements of linguistic incommensurability across different activities, this is in itself no reason to think that transfer through analogy cannot and does not occur.

When addressing Lave's study of the dieter – a study that makes up one of the practice-based arguments against transfer – we are dealing with a use of analogy that I suggest is integral to an apt description of what the dieter described by Lave achieved: making a *deductively* valid argument. This use of analogy is frequently overlooked, in so far as analogical argument is taken to appeal to a range of identical relations between objects in two phenomena, where the existence of the shared range of relations *inductively* supports the existence of a further shared property. While not made explicit, I suggest that this is the form of inference which Gentner, Ratterman & Forbus rely on in their understanding of inferential soundness when they describe analogy as "a one-to-one mapping from one domain representation (the base) into another (the target) that conveys that a system of relations that holds among the base objects also holds among the target objects, independently of any similarities among the object

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5   I am in full agreement with Gentner, Ratterman and Forbus (1993) when they suggest that analogy and metaphor are wrongly treated as largely unrelated phenomena in the learning literature. The relation between metaphor and analogy is explored by White (1996).

6   Of course, one can insist that "long" simply is not univocal. In that case, the challenge is to offer an account as to *why* the same word is used: the usage appears systematic, unlike words that are ambiguous by accident, such as "bank".

to which these relations apply" (1993, p. 526), and elements of structure mapping as "(2) *matching* the prior (base) analog with the target, and (3) *mapping* further inferences from the base to the target" (1993, p. 527). The inductive form of argument that I suggest above undoubtedly captures much structure mapping in scientific as well as learning contexts. There are, however, other logical forms that analogical arguments can exhibit, and to understand how analogy can support a *deductively* valid argument, we return to the treatment of triangles in Euclid.

As Gentner, Ratterman and Forbus (1993) concur, an analogy fundamentally sets up a model between objects or sets of objects, be they lines, triangles, waves or something far more complex. "Model" is by no means an unambiguous word across logic, mathematics and philosophy of science, and for the present purposes, we restrict ourselves to the relatively simple scale model.[7] A triangle can be modeled by another triangle with sides of various side lengths,[8] but invariant angles, as in two differently sized equilateral triangles used as models for one another. When such simple scale models are made, some properties will be invariant under modeling (angles) and some will not (e.g. color). This fact can be used to construct an argument from the model to what is modeled: We can infer nothing about the color of the model of a triangle, but we can, for instance, infer something about the length of its sides, given the length of one of them, combined with knowledge that the model is set up correctly. That the model is set up correctly, is, of course, *not* known in *most* scientific and learning contexts where modeling takes place, which is why the choice of properties from base to targets becomes a topic of central importance in an understanding of learning through analogy. For example, Kapon & diSessa (2012) rely on the idea that students are required to perceive *strong candidates for reality* in the base of the analogy (Clement & Brown, 2008; Harre, 1972) when using analogy to understand the target domain. Yet, *if* we know that the model is set up correctly concerning the properties we are interested in, there remain useful inferences we can make to extend our knowledge of these properties.

Maps and timelines serve as a case in point. The point of investigating triangles is that similarity in triangles is readily extended to geometrical figures *in general*, where two arbitrary figures will be geometrically similar if and only if every triangle inscribed on one figure can also be inscribed in the second. This fact is utilised in triangulation for map-making. The utility of maps relies on the invariance of geometrical properties under analogy: if the shortest line between two dots requires crossing a blue line, one can – given facts about the way colors represent topological reality – validly deduce features of what is being modeled: that the shortest route between two cities involves crossing a body of water. In short, reading maps is in fact doing logic and

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7   A model is understood here in the following sense: "Given two sets of objects, {$a_1, \ldots a_i, \ldots a_j, \ldots$} and {$b_1, \ldots b_i, \ldots b_j, \ldots$} and an operation R, then if ($\forall i$)($\forall j$) $a_i R a_j = b_i R b_j$, then either set of objects may be regarded as a model for the other" (White, 2010, p. 25)
8   More precisely, two triangles are analogous models of one another if ABC and A'B'C' with sides of lengths a,b,c and a',b',c' are similar if a/a' = b/b' = c/c'.

making inferences, and making maps is a process of creating models suited for such inferences. Another frequently used example of this is the inference from a timeline to the relation of events in time. While distance and time are different phenomena, they can be indirectly compared through an analogous relation between distances between objects (marks on a line, typically) and the temporal relation between events. From a correct timeline of Benjamin Franklin's life, we can immediately make deductively valid inferences about the relation between events in his life.

4. The pieces of transfer – what the dieter already knew

> *"… analogy is our best guide in all philosophical investiga-tions; and all discoveries, which were not made by mere accident, have been made by the help of it."*
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> *(Priestley, 1966, p. 14)*

In the following, we rely on our brief account of analogy to offer a different account of Lave's data related to the dieter. The simple scale models just explored do not account for the variety of uses of analogy in religious, philosophical, scientific or just plain folk use of language and reasoning. However, the modeling of one phenomenon on another is a basic and ubiquitous kind of cognitive operation that is identifiable across a range of otherwise different domains. It certainly drives sophisticated scientific developments, as when James Clerk Maxwell *transferred* results from a theory of heat to a different branch of physics, the theory of attraction at a distance. Such scientific uses of analogy are generally well explored, particularly since the publication of Hesse (1963). What I wish to underscore here is its commonality with more everyday and simple cognitive operations, such as that performed when reading a map and by Lave's dieter.

While one can easily overstate the case for analogy, there is much to suggest that the arithmetical discovery made by Lave's weight watcher was anything but, in Priestly's words, mere accident, or *purely* explicable as a matter of social context and activity. Rather, it seems that Lave in her observations of a weight watcher had come across a case of simple and successful analogical modeling in response to an arithmetical quandary. Not able to do the numerical operations ($3/4 \times 2/3$), the dieter devised a geometrical model of the guidelines. Inference from the model (three quadrants of two thirds of a cup of cottage cheese spread out on a table) to reality (or in this case, the instructions prescribed by the diet) was made, and inferences successfully drawn on the basis of the model. Appeal to the successful construction of a model and its subsequent use to make a valid inference can in this case offer a better account of Lave's data than her characterisation of the cognitive feat of the dieter.

Firstly, we can actually identify the piece that was likely transferred. With the proviso that we do not have relevant information from the learner himself, nor have we been able to consult textbooks relevant to time and place, there is much to

suggest that knowledge of the modeling of fractions by geometrical figures was central to the dieter's performance. That is, knowledge that fractions (target) can be modeled successfully with invariant mathematical properties by geometry (base), was successfully relied on by the dieter. This kind of modeling, frequently featuring illustrations of sliced cakes or pizzas, is a ubiquitous way of modeling fractions in instructional settings. Further, it is one that is frequently extended to basic arithmetical operations, such as multiplication of fractions, which is likely to be perceived as a unique achievement by the dieter, along with his use of ready-at-hand materials. We are likely to forget that this *is* successful modeling, due to its basic nature. Had we had the opportunity to talk to the dieter about his learning history in order to locate the piece that was transferred, we would have encouraged him to think back further in his learning and schooling history than his own suggestion of "a calculus course in college" (Lave, 1988, p. 165).

Secondly, we can account for the strong certainty that the dieter exhibits. Kokinov and French (2003) suggest that the perceived strength or plausibility of the analogical inference is a key issue in analogical reasoning. Several theories emphasise existing knowledge of the target domain (Kapon & diSessa, 2012) or pragmatic features of the learning situation (Holyoak & Thagard, 1989) in understanding this certainty. Kapon & diSessa (2012) suggest that a learner's existing knowledge of the target domain and recognition of this knowledge is of key importance when seeking to understand analogical modeling. With both scientists and learners, uncertainty about the source of the analogy is frequently the order of the day. The strong certainty on behalf of the dieter, once the method was devised, has two sources in our account: The first comes from a likely instructional setting during the dieter's schooling, where fractions were modeled with pictograms of sliced cake or pizza. Lave suggests that intuition and feeling are somehow in opposition to logic: "Authority of technology indicates exactitude, rationality and 'cold' logic which stands in mutually exclusive relations with intuition, feeling, and expression" (Lave, 1988, p. 125). Yet, *feelings* of obviousness are frequently appealed to in cognitive theories of learning. In the knowledge-in-pieces approach, these feelings are known as phenomenological primitives that develop "… when interacting with the physical world … [they] are recognized and evoked as whole, and they account for people's comfort with certain situations" (Kapon & diSessa, 2012, p. 265). The situation the dieter faced is likely to have evoked a recollection of physical modeling of fractions. A second source of explanation for the dieter's certainty comes from highly theoretical discussions of kinds of knowledge (most often, a priori knowledge) where feelings are identified as being close in kind to the kind of knowledge one possesses when one knowingly makes a deductively valid inference. In the theory of knowledge, fundamental kinds of knowledge are frequently appealed to in attempts to provide a foundation for further kinds of knowledge. To provide justification for such knowledge, appeals to a certain "evident lustre" (John Locke's expression) or "luminosity" (Williamson, 2000) or a certain kind

of immediacy and self-evidence are made.[9] In short, one has a phenomenological experience that "things must be so". There is similar, albeit anecdotal, evidence from logic tutors when they on rare occasions fail to convince students of the validity of simple inferences, such as *modus ponens*. Once their stock of examples is exhausted, tutors are frequently left making avowals of their feelings of certainty. One may of course have this phenomenological experience in many different settings, and the role of such feelings is a matter of controversy. Still, whatever the role of such feelings, there is widespread agreement that they are associated with claims of knowledge with the highest epistemic strength, such as knowledge of the certainty of a deductive inference. Making a successful inference with deductive strength fits the report of the dieter "*having got it*" much better than the dieter just *trying something on*. Unlike most learning situations, he already knew with certainty that sliced cottage cheese, cups and pizza are models of the fraction in question.

Finally, I note that Lave found it difficult to analyse the dialectical nature of the cottage cheese calculation because she insisted on placing problem solving in unique locations (in the head or on the shelf) and one "label[s] one element in a problem solving process as a 'calculation procedure', another as a 'checking procedure'" (1988, p. 164). Lave described the actions of the dieter in the following way: "The setting was part of the calculation process and the solution was simply the problem statement, enacted with the setting." (1988, p. 165). That is, Lave encouraged us to see something that arises only in a given, particular social and material context with no generality at work in the explanation. However, as I have shown, it is perfectly possible to make validly deductive arguments on the basis of a scale model. The dieter used what was available in his surroundings to make a simple, physical, scale model of the dietary recommendations, and immediately made a valid inference based on this model, seeing that the relevant features are invariant between the model and reality.

Modelling with physical reality is explored in a book length study by Levi (2009). He seizes on George Polya's remark on Archimedes' discovery of integral calculus that "one of the greatest mathematical discoveries of all time was guided by physical intuition" (Pólya, 1954, p. 154) and demonstrates its relevance to a range of mathematical problems. What Levi calls physical arguments are "not rigorous … but can be a tool of discovery and of intuitive insight – the two steps that precede [mathematical] rigour" (2009, p. 3). As Levi investigates, many problems treatable in mathematics turn out to have geometric and mechanical solutions that exhibit the virtue of being readily

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9   Possibly the most prominent example in logic is Gottlob Frege's appeal to "self-evidence" in his treatment of logical axioms. These played a role in his attempt at providing a logical foundation for arithmetic and served as a point of departure for Wittgenstein's early work on logic: "… it is remarkable that so exact a thinker as Frege should have appealed to the degree of self-evidence as the criterion of a logical proposition" (Wittgenstein, 1922, p. 6.1271).

accessible, requiring little computation and leading to new discoveries. For example, it is possible to construct what Levi calls a kinematic proof of the Pythagorean theorem, relying on a prism-shaped fish tank and the fact that still water in a resting container, with no disturbances, remains at rest. In direct parallel with Lave's description of the dieter, Levi emphasises a mechanical solution: "Note that we did not have to build the fish tank, not even in the thought experiment; rather, we can imagine the prism of water embedded in a larger body of water" (2009, p. 11). Of course, Levi does not see such explorations as a repudiation of universal kinds of reasoning such as mathematics. He suggests that examples of physical reasoning can aid mathematics instruction. Further, he ponders the relation between physics and mathematics, given that the *same* mathematical content can be represented in different ways – physically as well as mathematically – and that these different representations have different strengths *vis a vis* intuitive appeal as well as transferability.

Studies like Lave's highlight the fact that modeling and making inferences cut across any "elite – just plain folk" dichotomy, underscoring the various ways that mathematical knowledge can be represented, and accordingly, transferred. As suggested above, we shall have to look in a different place for learning that has transferred than mathematical textbook material and classroom practices that Lave naturally considered. Along the lines of diSessa and Wagner's (2005) coordination class theory, understanding learning transfer would require careful analysis of the dieter's learning history and existing knowledge, as well as the formal kinds of reasoning surveyed above.

5. Concluding remarks

Having reread Lave's highly influential book, I have identified and confronted two strands of argument against cognitive approaches to transfer that are expressed in, and arise from, her work. Firstly, I have defused a concept-based criticism through an exploration of the phenomenon of analogically related concept use across different, incommensurable domains. Secondly, I have confronted a practice-based case against transfer, where an account of inferences based on scale models plays a key role. I have argued that one of the on-the-go arithmetical operations reported by Lave appears to rely on a highly universal kind of knowledge: knowledge of logic and modeling. By emphasising these aspects of concept use and the importance of identifying "small pieces" of knowledge displayed in modeling, I have drawn on key themes in the transfer-in-pieces approach to understanding learning. I maintain that relying on conceptual resources from a cognitive approach to learning offers a better analysis of Lave's data than what she could offer. My alternate analysis of Lave's data has only treated one specific case, among the many that Lave relied on. Analyses of these cases will likely differ from the one that has been explored in this paper, and some are already available. Some suggest that Lave did indeed document the existence of transferable mathematical skills, such as estimation (Reed, 2012), or that she looked for a

certain kind of mathematical transfer where none ought to be found, as also suggested to her by her informants (Greiffenhagen & Sharrock, 2008).

Lave's work played a significant role in shaping a field that continues to solidify into different "approaches" to, or "theories" of, learning. Popular and scholarly treatments identify behaviorist, cognitivist, constructivist, constructionist, socio-cultural, situated, participationist and other approaches (Dohn et al., 2020; Reed, 2012; Selwyn, 2017). However we assess the details of Lave's empirical material and its analysis in her book, her work ultimately deserves praise and recognition on three counts. Firstly, it has played a key role in shaping territories on the contemporary map of learning theories. Secondly, her studies of transfer in settings quite different from that of instruction is in keeping with the overarching reason that learning transfer has long remained a key topic in theories of learning and education. Speaking of learning transfer is at least one way of conceptualising what we also hope to achieve from attending school, college or university: lasting skills and knowledge that can be utilised in new and often unforeseen settings, outside formal learning contexts. Many influential studies of transfer continue to study transfer *within* contexts of formal learning, such as school and university (Engle et al., 2011; Lobato, 2008). Finally, her work displayed a willingness to enter into fierce, but also fruitful debate with other areas on the map, in particular cognitive approaches to learning. Keeping a central aspect of learning education, such as transfer, in focus and keenly debating competing theories, is a scientific virtue that deserves emulation.

Acknowledgements

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